MAGNET LAMINATION EDDY CURRENTS REEXAMINED

Booster Technical Note

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The first analysis of this problem (Tech. Note no. 8, 2/12/86) consisted of the solution to the 1-dimensional magnetic diffusion equation with constant parameters and with the triangular time waveform expanded in a Fourier series; the analytic series solution was evaluated numerically. Subsequently an analytical solution in which the linear time dependence appears explicitly, rather than as an expansion, was obtained and was presented at the Booster review in June. Since this was not published, a summary of it is included as an appendix. The important findings are that a steady-state condition is attained with a time constant $\tau = d^2\mu/(4\pi^2\rho)$, and that the mean field lags the surface field by an amount $\delta t = d^2\mu/(12\rho)$.

It was pointed out by John Galayda (of NSLS) that the currents at the edges of the laminations may introduce harmonics in the magnet gap; his conclusion was that the effects are unimportant for the NSLS.

A 2-dimensional analysis using the approach given in the appendix was first completed. The magnetic diffusion equation for the field in the lamination is

$$\partial^2 H_y/\partial x^2 + \partial^2 H_y/\partial z^2 = (\mu/\rho)\partial H_y/\partial t \quad \text{(MKS units)} \tag{1}$$

The coordinate system has z in the beam direction, y is vertical, and x transverse. For a lamination of thickess d and pole width w, the solution to (1) is given by eqn (2). It assumes $\bar{H}$ = at for the ramp up and $\bar{H}$ = -at for the down ramp, where $\bar{H}$ is the spatial average field.

$$\begin{aligned} H = {} & \mu a/4\rho[x^2+z^2-(d^2+w^2)/12]+at \\ & -\mu a/(2\pi^2)[w^2\Sigma(-1)^n/(n^2)\exp(-t/\tau_n)+d^2\Sigma(-1)^m/(m^2)\exp(-t/\tau_m)] \end{aligned} \tag{2}$$

where $\tau_n = (\mu/\rho)(w/2n\pi)^2$ and $\tau_m = (\mu/\rho)(d/2n\pi)^2$, n and m > 0. It is assumed that both τ_n and τ_m are much less than half the period, i.e. $\tau \ll T/2$.

The pole face width in the Booster is about 250 mm and the lamination thickness is about 1 mm, so τ_n is about 62500 times τ_m. In the earlier work it was assumed that a τ_m of about 0.4 msec was desired, which would mean τ_n = 25 second. Since this is much greater than T/2, this solution is inappropriate.

The appropriate solution follows the methodology of TN no.8, i.e., expand the waveform in a Fourier series. The solution is given by equation (3).

$$\begin{aligned} H = \frac{A}{2} - \sum_{n} \frac{2A}{n^2\pi^2} \Bigg\{ & \left[\frac{d\cosh(k_1 z)}{\sinh(k_1 d/2)} + \frac{w\cosh(k_1 x)}{\sinh(k_1 w/2)}\right] k_1 e^{in\omega t} \\ & + \left[\frac{d\cosh(k_2 z)}{\sinh(k_2 z)} + \frac{w\cosh(k_2 x)}{\sinh(k_2 w/2)}\right] k_2 e^{-in\omega t}\Bigg\}, \quad n \text{ odd} \end{aligned} \tag{3}$$

where $k_1 = (1+i)/\delta_n$, $k_2 = (-1+i)/\delta_n$ and $\delta_n = \sqrt{[2\rho/(n\omega\mu)]}$ is the skin depth of the n^{th} harmonic.

Equation (3) shows that the crossing current is confined to a region at the edge of the lamination thickness of about a skin depth of the fundamental, $n = 1$, i.e., about 1.5 mm in a typical material at proton cycle frequency, or 3 mm at the heavy ion cycle frequency (2 Hz). This current sheet on the sides of the poles is very close to the exciting coils and hence will not perturb the field in the gap except for a small decrease in the fundamental.

APPENDIX

For eddy currents in the laminations, assume the field in the laminations is parallel to their surface. Look for the solution to the one-dimensional diffusion equation:

$$\frac{\partial^2 H}{\partial x^2} = \frac{\mu}{\rho}\frac{\partial H}{\partial t} \qquad \text{(MKS units)}$$

The form for the solution with constant rate of change and equal field on both sides of the lamination is

$$H = b\cos(\sqrt{\lambda}\,x)\,e^{-\lambda t\rho/\mu} + \frac{\mu}{\rho}\frac{ax^2}{2} + at + c$$

Assume that the field in the magnet gap is equal to the mean field $\bar{H}$ in the lamination, i.e. $\bar{H} = \frac{1}{d}\int_{-d/2}^{d/2} H\,dx$ where d is the lamination thickness.

Require $\bar{H} = at$ for the ramp up. The field at the surface of the lamination is then

$$H_s = at + \frac{ad^2}{12}\frac{\mu}{\rho}\left[1 - 2e^{-t/\tau}\right]$$

where $\tau = \frac{d^2}{4\pi^2}\frac{\mu}{\rho}$. This solution assumes the time constant tau is $<< \frac{1}{2}T$ where T is the period.

At a time $t < T/2$, but also large enough that the transient has dropped to a negligible value,

$$H_s = at + \frac{\mu a d^2}{12\rho}$$

At a some what later time, $t + \delta t$, $\bar{H} = a(t+\delta t)$ has the same value. This time delay is

$$\delta t = \frac{d^2}{12}\frac{\mu}{\rho}$$

δt is the time by which the mean field lags the surface field. It is proportional to tau, i.e.,

$$\tau = \frac{3}{\pi^2}\,\delta t$$

The delay can also be expressed in terms of the skin depth of the fundamental, $\delta = \sqrt{\frac{2\rho}{\mu\omega}}$. For the present case of T = 0.1 second,

$$\delta t = 2.653\left(\frac{d}{\delta}\right)^2 \text{ millisecond}$$

To compare eddy current effects among the different materials, a thickness was computed for each which has a δt of 1.24 msec, and hence a τ of 0.377 msec. The δt was selected to correspond to the time estimated for transients in the vacuum chamber to damp.

mat'l	M19	M22	M27	M36	M43	iron	Algny #2	Algny#5
d, mils	41.1	42.7	42.7	38.5	35.0	30.4	42.8	32.4
We, W/m	1.4	1.6	1.9	2.1	2.1	2.7	2.5	1.1
Wh, W/m	5.2	6.0	7.6	7.9	8.5	19.	(4.1)	(1.6)

We is the eddy current loss, watts/meter and Wh is the hysteretic loss. We is calculated from $\rho\left(\frac{\partial H}{\partial x}\right)^2$ averaged over the lamination thickness, assuming a peak to peak amplitude of 2.5 kG and assuming a 100% duty cycle. Wh for the M series steels is extrapolated from Metals Handbook data for 60 Hz and 10 to 15 kG peak amplitude. These were extrapolated to zero thickness to eliminate the eddy current component, then extrapolated to 2.5 kG peak to peak amplitude assuming a $B^{2.23}$ dependence. The value for soft iron is for .008% carbon given in the Metals Handbook. The values for the Allegheny materials are estimates based on M22 and M27, scaled proportionately to Hc.